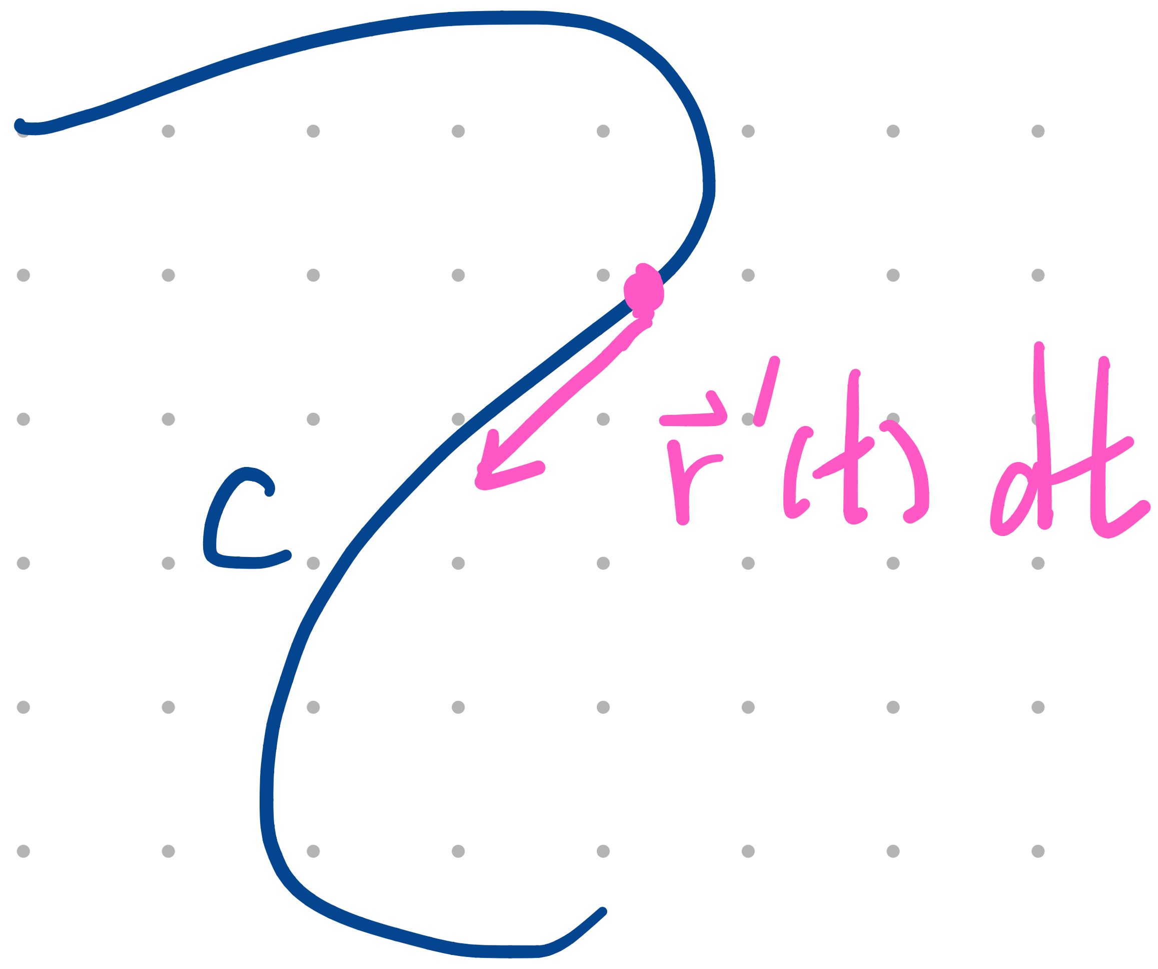
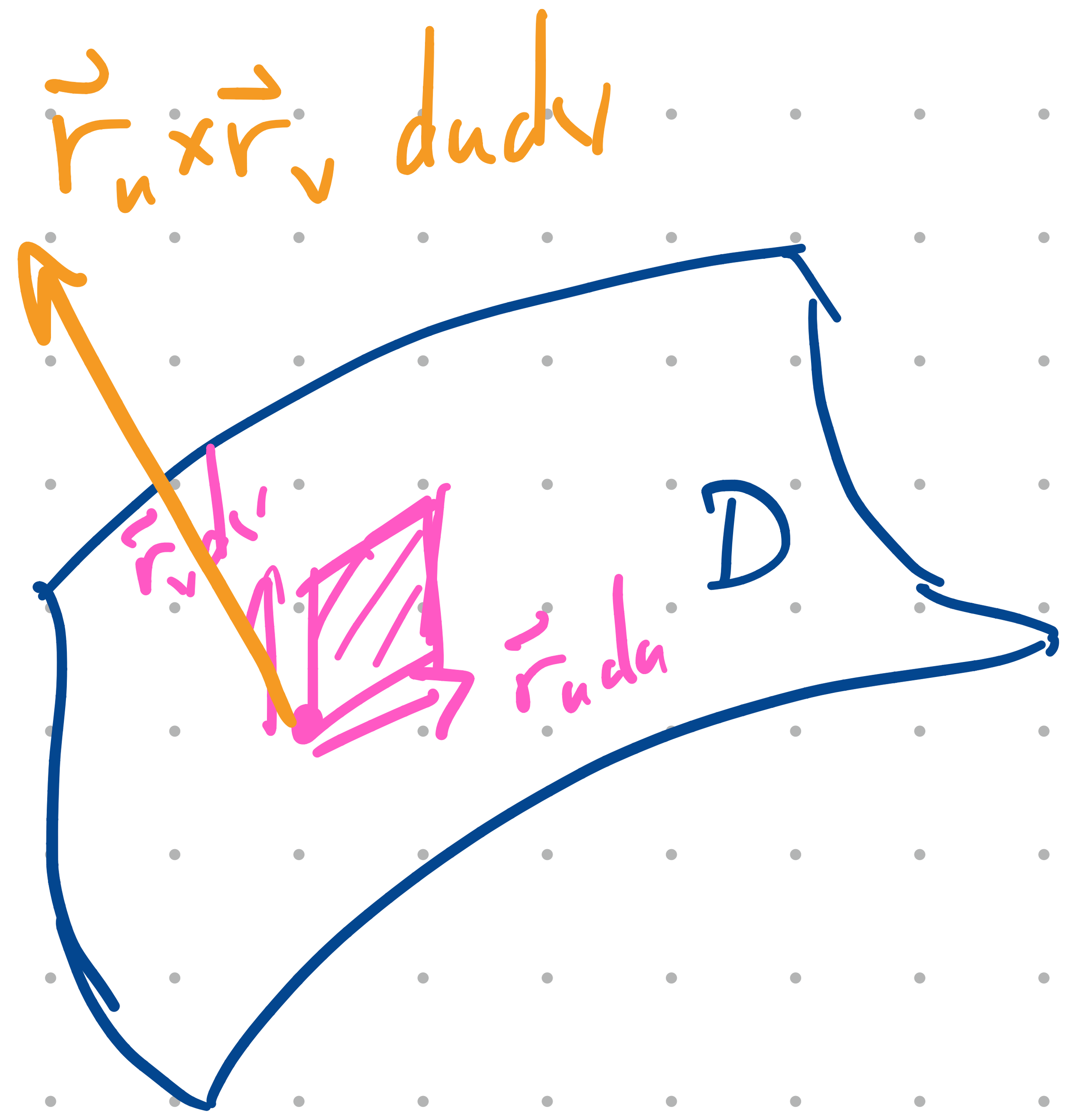




$$\int_C f \; \underbrace{ds}_{|\vec{r}\,'(t)|\,dt} \qquad\qquad \iint_D f \; \underbrace{dS}_{|\vec{r}_u \times \vec{r}_v|\,du\,dv}$$

for the above integrals, we discarded the information about direction.

Work:

$$\int_C \left(\text{comp}_{\vec{r}\,'} \vec{F}\right) ds = \int_{t_0}^{t_1} \vec{F} \cdot \underbrace{\vec{T}}_{=\frac{\vec{r}\,'(t)}{\cancel{|\vec{r}\,'(t)|}}} \; \cancel{|\vec{r}\,'(t)|}\,dt$$

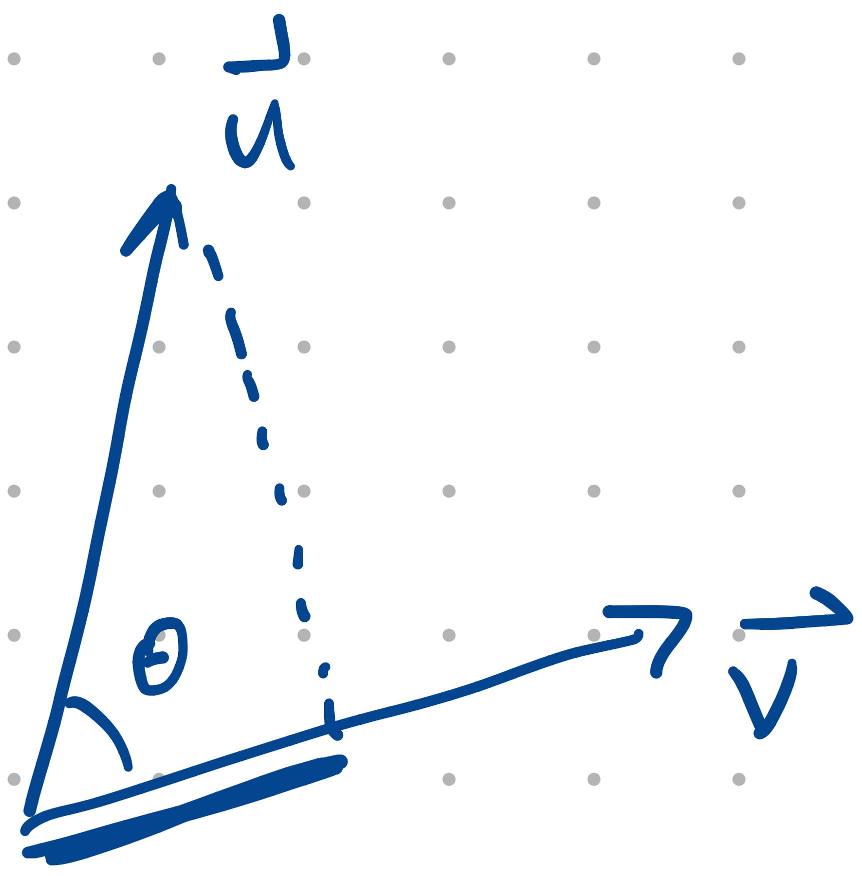
Flux:

$$\iint_D \left(\text{comp}_{\vec{n}} \vec{F}\right) dS = \iint_D \vec{F} \cdot \underbrace{\hat{n}}_{\text{unit normal}} \; dS$$

$$= \iint_{u,v \text{ region}} \vec{F} \cdot \frac{\vec{r}_u \times \vec{r}_v}{\cancel{|\vec{r}_u \times \vec{r}_v|}} \; \cancel{|\vec{r}_u \times \vec{r}_v|}\,du\,dv$$

⚠ Work and flux integrals care about the orientation of the curve or surface respectively.

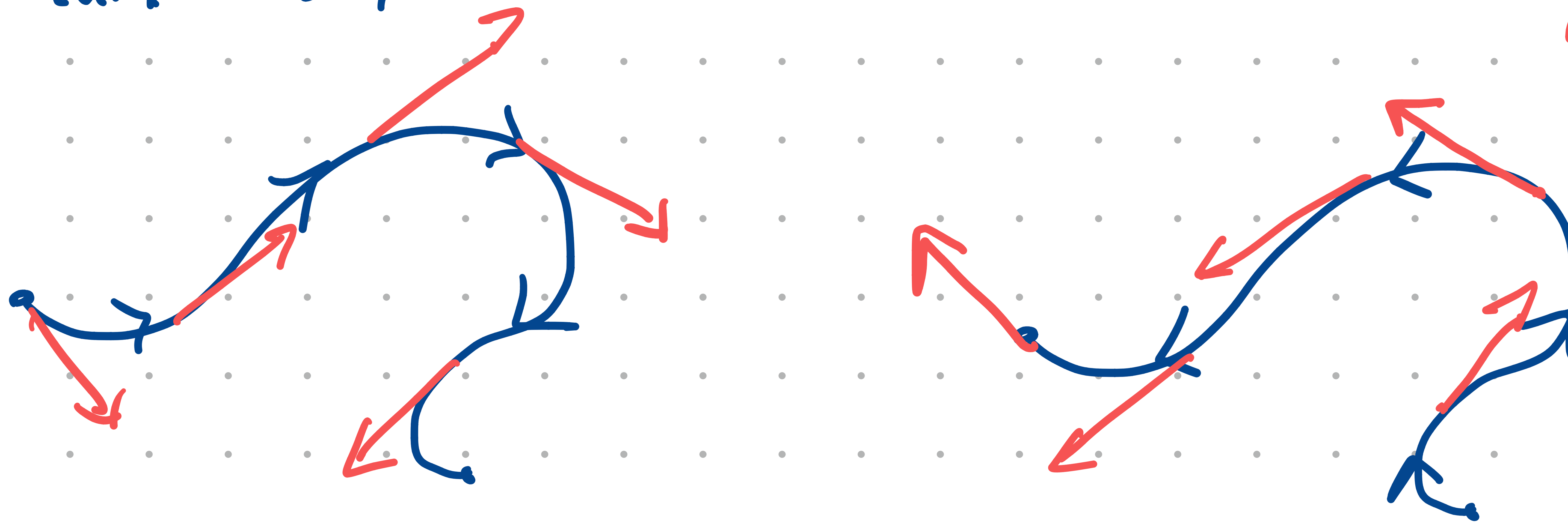
$$\mathrm{comp}_{\vec{v}}\,\vec{u} = |\vec{u}|\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}$$



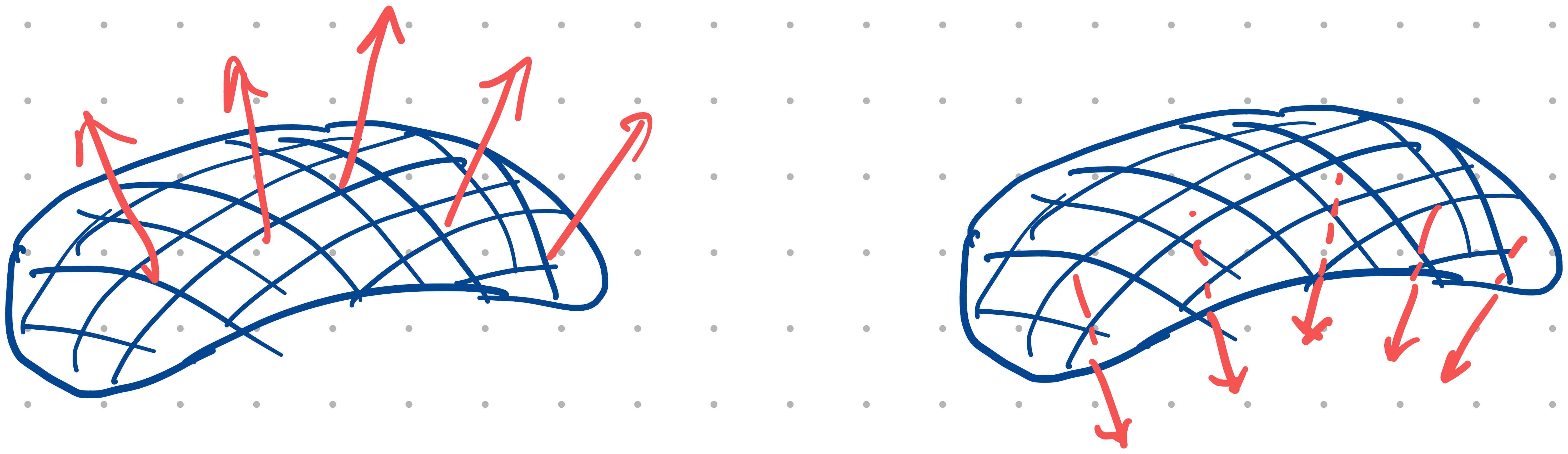
$$\left(\text{If } \vec{u} = \vec{F},\quad \vec{v} = \vec{r}\,'(t),\quad \vec{F}\cdot\boxed{\frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}}\right)$$

unit tangent $\vec{T}$

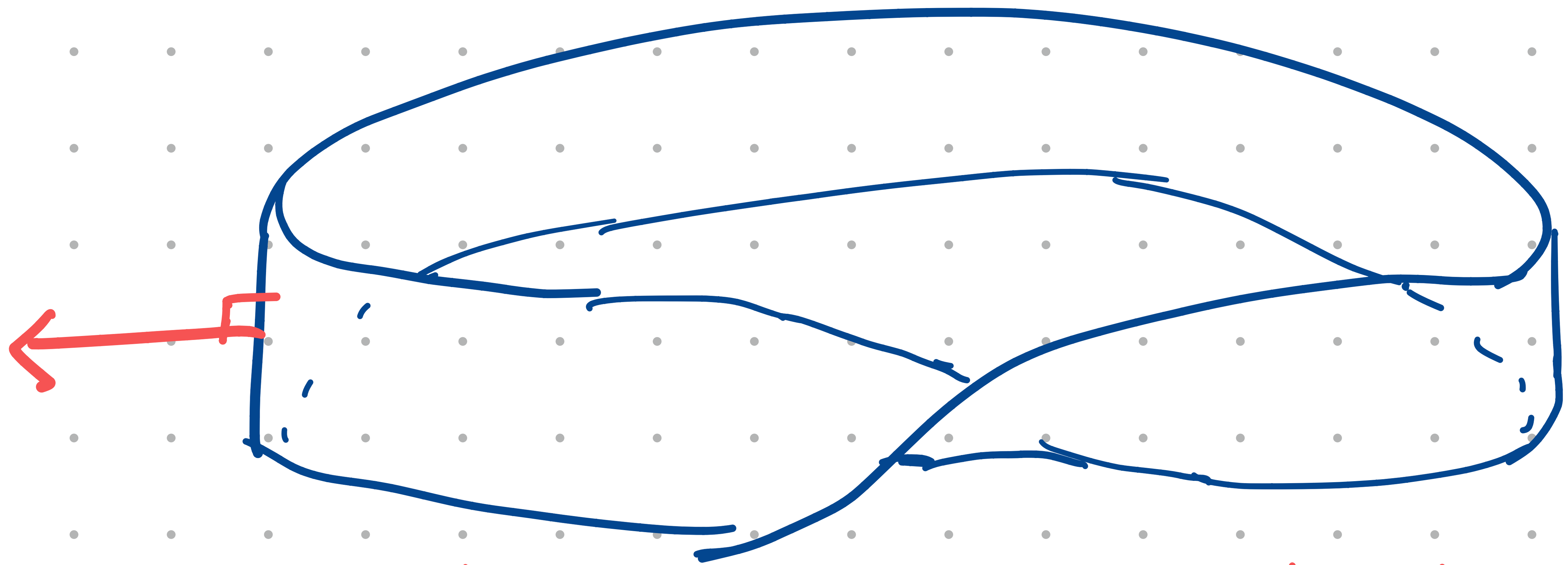
A curve has two orientations: i.e. two choices of $\vec{T}$ unit tang.



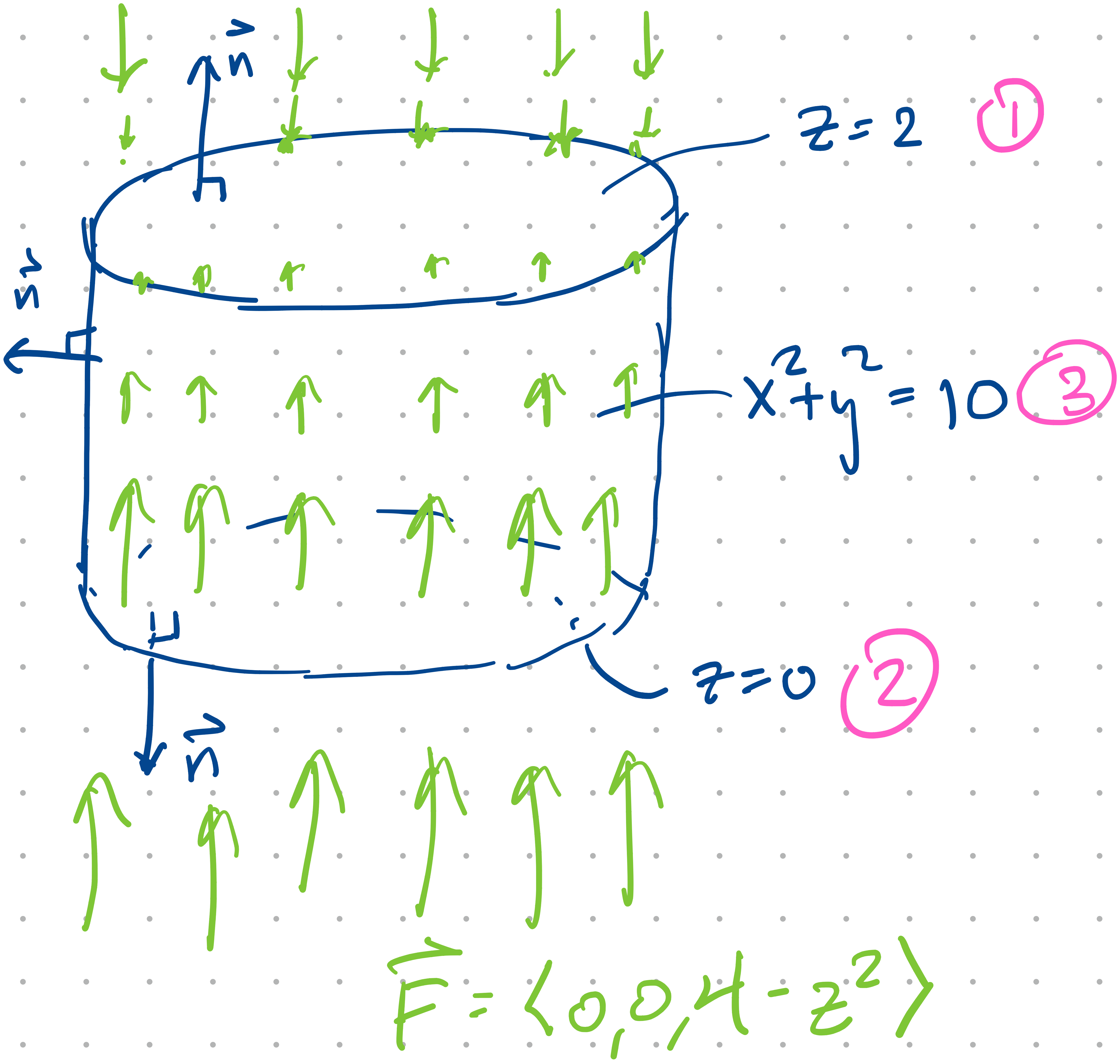
An orientable* surface has two orientations i.e. two choices of $\vec{n}$ unit normal



*An example of a non-orientable surface:



It is impossible to pick a consistent unit normal vector for this surface



$\langle u,v,2\rangle$

uv region: $u^2+v^2\le 10$

① $\displaystyle\iint_{\text{top}} \vec{F}\cdot d\vec{S} = \iint_{u^2+v^2\le 10} \langle 0,0,4-2^2\rangle \cdot \left(\langle 1,0,0\rangle \times \langle 0,1,0\rangle\right) du\,dv$

$\displaystyle = \iint_{u^2+v^2\le 10} \vec{0}\cdot\langle 0,0,1\rangle\, du\,dv = 0$

excessive to do it this way, but done for the sake of illustration.

(2) $$\iint_{\text{bottom}} \vec{F} \cdot d\vec{S} = \iint_{\text{bottom}} \langle 0,0,4\rangle \cdot \vec{n}\, dS$$

$$= \iint_{\text{bottom}} \langle 0,0,4\rangle \cdot \langle 0,0,-1\rangle\, dS$$

$$= -4 \iint_{\text{bottom}} 1\, dS = -4(\pi 10)$$

$$= \boxed{-40\pi}$$

(3) Answer is 0 b/c $\vec{F} \cdot \vec{n} = 0$.

<u>Rmk</u>. Although $\vec{r}_u \times \vec{r}_v$ is a fine general way of producing a normal vector, in some situations parametrization may not be necessary (nor convenient).

i.e.

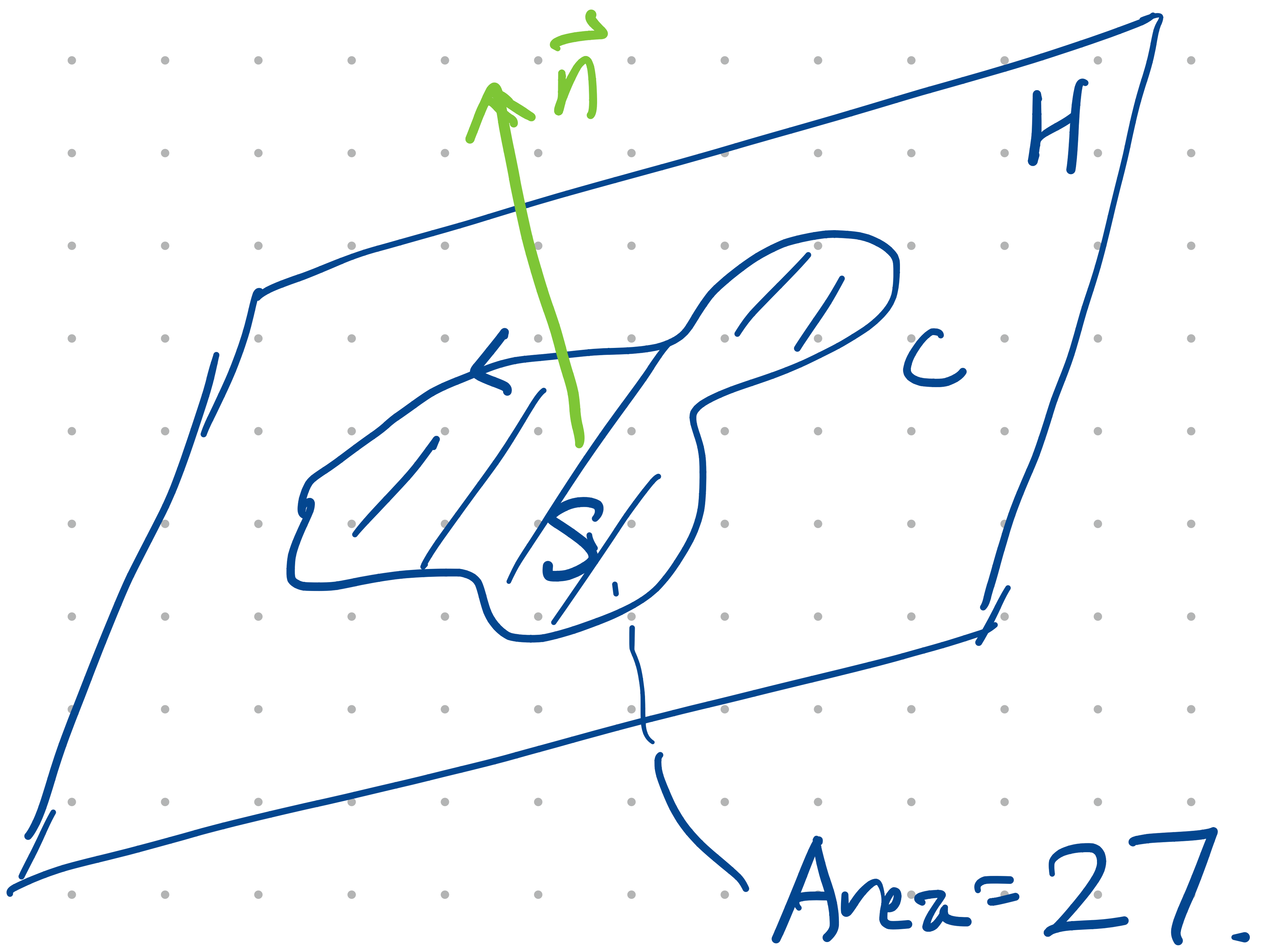
$$\iint_{\text{surface}} \vec{F} \cdot \hat{n} \, dS \qquad \iint_{\substack{uv\text{-region} \\ (\text{via param.})}} \vec{F} \cdot (\vec{r}_u \times \vec{r}_v) \, du \, dv$$

sometimes it might be easier to just directly use this expression (and compute $\hat{n}$ in <u>other ways</u>)

Problem: Let H be the plane $3x+4y+z=7$.

Let $\vec{F} = \langle 0, x, 0 \rangle$

Let C be a closed curve on the plane H,



CCW when viewed from above

Question: What is $\oint_C \vec{F} \cdot d\vec{r}$?

Sol: By Stokes:

$$\ldots = \iint_S (\nabla \times \vec{F}) \cdot d\vec{S} = \iint_S \langle 0,0,1 \rangle \cdot \vec{n} \, dS$$

oriented according to RHR

$$= \iint_S \langle 0,0,1\rangle \cdot \frac{\langle 3,4,1\rangle}{|\langle 3,4,1\rangle|}\, dS$$

this is upwards, so we picked the right unit normal.

$$= \iint_S \frac{1}{\sqrt{26}}\, dS$$

$$= \frac{1}{\sqrt{26}} \underbrace{\iint_S 1\, dS}_{\text{surface area of } S} = \boxed{27/\sqrt{26}}.$$